1.5 Graphical Description of Accelerated Motion

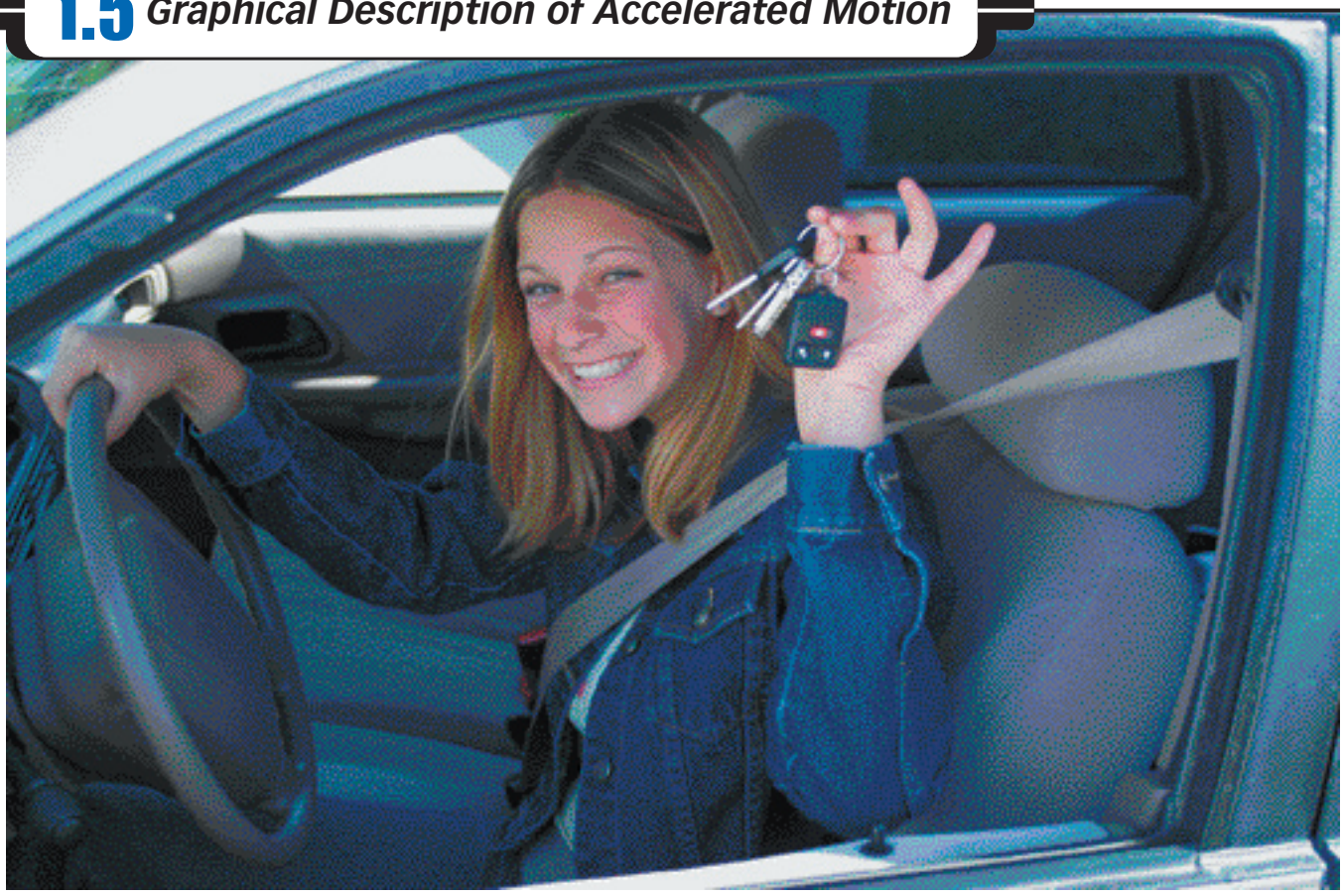


Figure B1.32: Most of your vehicle trips can be described as non-uniform motion.

Think back to the last time you made a short trip in a vehicle. Maybe you were running a quick errand, like returning a DVD rental; or maybe you were on your way to your part-time job, hockey practice, or music lesson. What parts of your trip could be described as being uniform motion? To answer this question, you would have to think about the parts of the trip that involved travelling at a constant speed in a straight line.

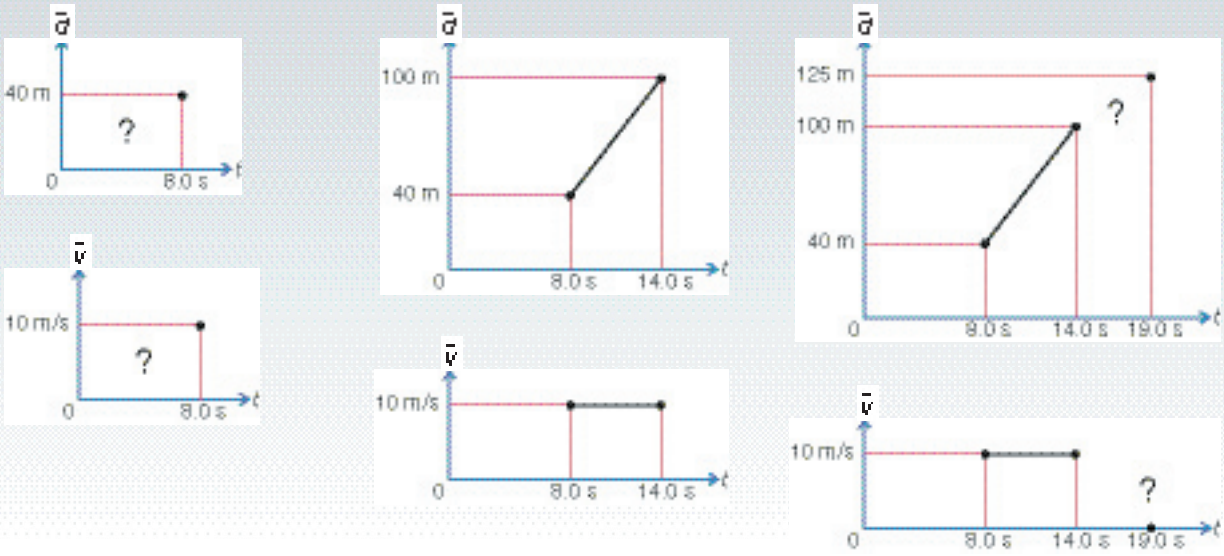
Uniform motion usually occurs during highway driving because the roadway tends to have long, straight sections in which the vehicle is travelling at a constant speed. So, if your short trip involved even ten minutes of travel on a highway, uniform motion likely occurred.

Uniform motion is more difficult to maintain in populated areas due to the increased number of intersections. Since a driver's route may involve turning at intersections, the straight sections of roadway tend to be much shorter, preventing a driver from travelling in a straight line. The most significant effect of the increased number of intersections is that a driver is required to slow down and speed up much more frequently than while driving on a highway. If you are stopped at an intersection, once the light turns green, you have to speed up from rest to some new speed. You may be able to achieve uniform motion at your new speed, but this would likely last only a few seconds. It is inevitable that you will have to adjust your speed due to traffic and other conditions, like coming to a stop at another red light or at a stop sign.

As you leave the first intersection, the vehicle is not travelling with uniform motion because it is speeding up. If you travel 40 m in 8.0 s, what shape will the position–time graph and velocity–time graph have for this part of the motion?

If the road is straight and traffic conditions allow for it, you may be able to travel with uniform motion for a brief time between intersections. If you were able to travel at 10 m/s from a position 40 m from the first intersection to a position 100 m away, the graphs might look like this.

When your position is 100 m from the first intersection, you start to slow down because the light at the next intersection is about to turn red. Suppose it takes you 5.0 s and an additional 25 m to come to a stop. What is the shape of the position–time graph and velocity–time graph for this time interval?



Practice

23. Refer to the graphs for the middle section of the motion, where you were travelling with uniform motion.
    a. Use the information on the position–time graph to verify that the velocity was 10 m/s during the period of uniform motion.
    b. Use the information on the velocity–time graph to verify that you travelled 60 m during the period of uniform motion.

A Graphical Description of Accelerated Motion

One of the best ways to answer the questions posed by the previous graphs is to collect data for an object that is speeding up and slowing down and then actually build the graphs yourself. The benefit of this approach is that you will be able to practise linking a description of motion to the shape of a graph. This method will also establish a foundation for the equations that come from these graphs. The next investigations will give you this opportunity.

Investigation

Accelerated Motion—Speeding Up

Purpose

You will collect data for a dynamics cart rolling down a ramp. Will the velocity of the cart be constant? What will be the shape of the position–time graph? What will be the shape of the velocity–time graph?

Prediction

Make predictions to the questions posed in the purpose.

Materials

- ticker tape timer and ticker tape (or motion sensor)
- dynamics cart
- masking tape
- metre-stick
- table with two blocks of wood under the legs at one end (or a 2-m board with one end raised)

Procedure

Two procedures are given: one for using a ticker tape timer and another for using a motion sensor. Follow the appropriate procedure for your apparatus.

Using a Ticker Tape Timer

step 1: Set up the apparatus as shown in Figure B1.33.

Figure B1.33: Don't forget to put the blocks of wood under the table legs at one end.

Science Skills

- ✓ Performing and Recording
- ✓ Analyzing and Interpreting

step 2: Remove a length of ticker tape that is slightly shorter than the length of the path of the dynamics cart on the tabletop. Carefully unravel the ticker tape and thread it through the timer, with the loose end of the tape behind the timer.

step 3: Have a partner stand at the end of the table so the cart doesn't fall to the floor.

step 4: Release the cart; then turn on the ticker tape timer. Turn off the timer once the cart reaches the end of its path.

step 5: Stretch out the ticker tape on the table, and tape down both ends. Select a clear dot at the beginning to be the reference point. Label this dot with an arrow, and mark it as 0.

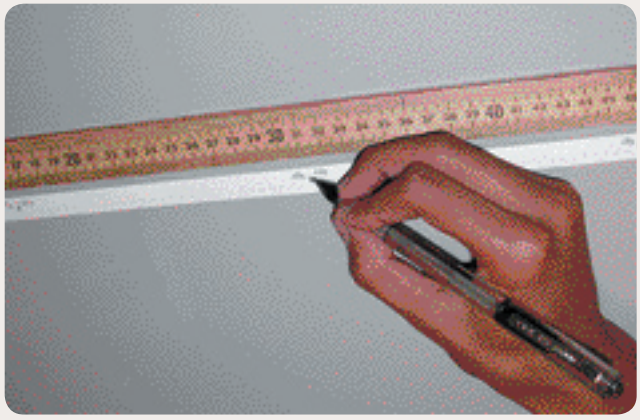


Figure B1.34: Mark a clear dot as your reference point and every thirtieth dot thereafter.

step 6: Your timer likely makes 60 dots per second; therefore, every thirtieth dot will be used to indicate a half-second. (Confirm this with your teacher.) Start counting the dots from the reference point. Label the thirtieth dot with an arrow, and mark it as 0.50 s. Continue this process until you have labelled every thirtieth dot on the ticker tape (as shown in Figure B1.34).

step 7: Copy and complete a table like the following. In the Time column, enter the times for every half-second.

Time (s)	Position (cm)	Average Velocity (cm/s)	Midpoint Time (s)
0			
0.50			
1.00			
1.50			

An alternative data collection method is to set up this table in a spreadsheet. Arrange the columns as shown in the sample table. Simply leave a blank line between each time entry to record your values of average velocity and midpoint time.

step 8: Align the end of the metre-stick with the reference dot, and record the position of the cart at every 0.5-s interval until you reach the end of the tape.

Using a Motion Sensor

step 1: Set up the apparatus as shown in Figure B1.35.

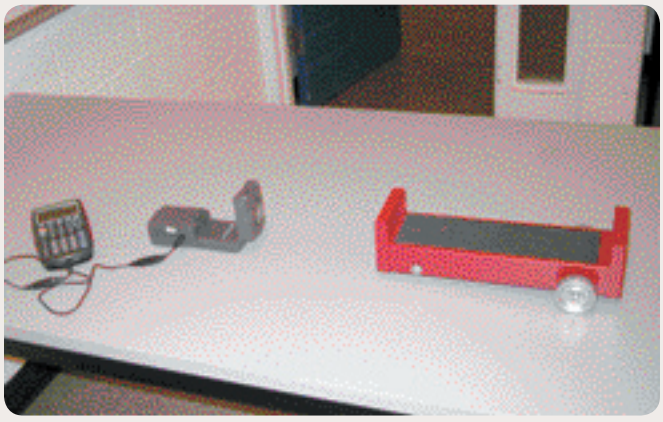


Figure B1.35: Remember, put blocks of wood under the table legs at one end.

step 2: Follow the manufacturer's recommendations for your particular motion sensor so it is able to collect data from the cart. Remember to have your partner catch the cart at the end of the table so it doesn't fall to the floor.

step 3: Set up a table similar to the sample table provided. The sensor will likely collect more data than you can use, so you will have to sample the data by recording times and the corresponding positions at 0.50-s intervals. Enter the values that are as close to every half-second as possible and their corresponding positions. If you are using a spreadsheet, leave a blank line between each entry to record your values of average velocity and midpoint time.

Analysis

1. Use the same procedure from the analysis of the UMD data in Lesson 1.4 to calculate the average velocity and the midpoint time for each 0.50-s time interval.
2. Use the first two columns of your table to plot a position–time graph. Draw a smooth curve that fits the data closely.
3. Use the last two columns of your table to plot a velocity–time graph. Use a ruler to draw the best-fit line.
4. Calculate the slope of the best-fit line on the velocity–time graph. Be sure to include units.
5. Describe the shape of the position–time graph. What characteristics of the shape of this graph describe the motion of the cart down the ramp?
6. Describe the shape of the velocity–time graph. How does the shape of this graph describe the motion of the cart down the ramp?

Evaluation

7. Do the results of this experiment support the shapes of the graphs you predicted in your hypothesis? Answer by comparing the predicted shapes of the two graphs with the actual shapes of the graphs.

Acceleration

Clearly, the graphs that describe an object that is speeding up are quite different from the graphs that describe an object that is moving with uniform motion. Both position–time graphs and velocity–time graphs can describe motion that involves increasing velocity. However, they tell this story in slightly different ways. Figure B1.36 provides a sample of what the graphs from the previous investigation might have looked like.

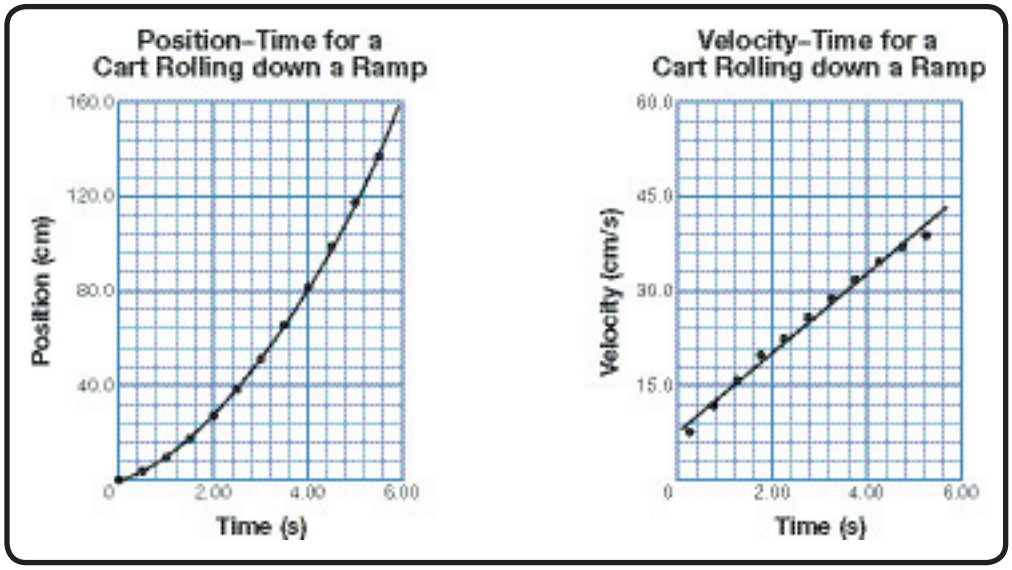


Figure B1.36: These are sample graphs from "Accelerated Motion—Speeding Up."

Notice that the velocity–time graph is a straight line; so, it is possible to describe the pattern of changing velocities with a single slope calculation. The slope of a velocity–time graph is called **acceleration**. Calculating the rise of this graph will always equal the difference between the velocity at the beginning of the time interval (initial velocity) and the velocity at the end of the time interval (final velocity). The run will always equal the time interval (change in time). This is why the equation is usually written like the one on the right.

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{change in time}}$$

$$\text{acceleration} = \frac{\text{final velocity} - \text{initial velocity}}{\text{change in time}}$$

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

acceleration: the change in velocity over a specific time interval

Example Problem 1.10

Calculate the acceleration from the sample data displayed in the graph in Figure B1.36.

Solution

To determine acceleration from the sample data, you must choose two points on the best-fit line. Here, the two points chosen are (1.20 s, 15.0 cm/s) and (5.00 s, 39.0 cm/s).

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$= \frac{39.0 \text{ cm/s} - 15.0 \text{ cm/s}}{5.00 \text{ s} - 1.20 \text{ s}}$$

$$= 6.32 \text{ cm/s/s or } 6.32 \text{ cm/s}^2$$

Note that in the solution to the sample problem, the units for acceleration can be written as cm/s/s or as cm/s^2.

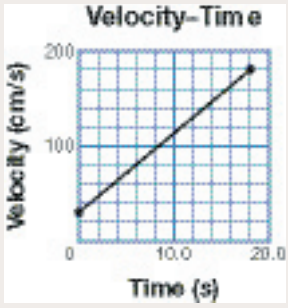
Not all non-uniform motion produces a velocity–time graph like the one from the previous investigation. This is an example of a special type of motion called **uniform acceleration**.

uniform acceleration: acceleration described by a sloping, straight line on a velocity–time graph

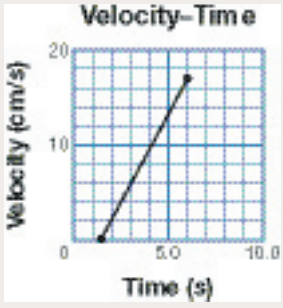
Practice

24. Determine the acceleration using the information provided by each of the following graphs.

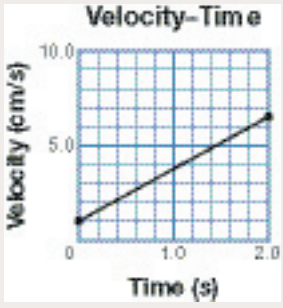
a.



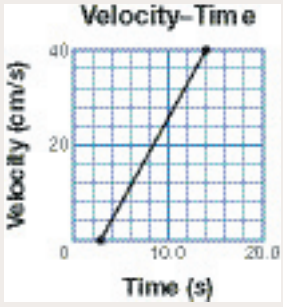
b.



c.



d.



Hints for Problem Solving

Most problem-solving applications involve situations where the acceleration may not be perfectly uniform all the time. In these cases, the term *average acceleration* is often used to remind you that your use of the acceleration equation is just averaging the changes in velocity over a specific time interval.

The techniques for rearranging equations can be applied to the acceleration equation. However, when solving for time, drop all the vector notations. This is because the multiplication or division of vectors goes beyond high school mathematics. The vector notation can also be dropped if the question has the velocity and acceleration vectors pointing in the same direction and if the question is not specifically asking you to determine a direction.

These hints for problem solving are illustrated in the example problems that follow.

Example Problem 1.11

A car travelling 70 km/h east changes its velocity to 90 km/h east in 4.5 s. Determine the magnitude and direction of the average acceleration of the car in m/s^2.

Solution

Since this question asks for a direction, include vector notation and a sign convention.

W E
\- +

$$\vec{v}_i = 70 \text{ km/h [E]}$$
$$= +70 \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1000 \text{ m}}{1 \cancel{\text{km}}} \times \frac{1 \cancel{\text{h}}}{3600 \text{ s}}$$
$$= +19.\overline{4} \text{ m/s}$$

$$\vec{v}_f = 90 \text{ km/h [E]}$$
$$= +90 \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1000 \text{ m}}{1 \cancel{\text{km}}} \times \frac{1 \cancel{\text{h}}}{3600 \text{ s}}$$
$$= +25 \text{ m/s}$$

$$\Delta t = 4.5 \text{ s}$$

$$\vec{a} = ?$$

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$
$$= \frac{(+25 \text{ m/s}) - (+19.\overline{4} \text{ m/s})}{4.5 \text{ s}}$$
$$= +1.2 \text{ m/s}^2$$

The average acceleration of the car is 1.2 m/s^2[E].

Example Problem 1.12

A sprinter on a high school track team can achieve an average acceleration of 3.0 m/s^2 from rest to his maximum velocity of 11.0 m/s.

a. Calculate the time taken by the sprinter to reach his maximum velocity.

b. The coach explained during training that the sprinter can improve his performance if he can increase his acceleration during the first phase of the race. Even if his maximum velocity remained the same, a higher rate of acceleration would give him better results. Explain this thinking by referring to the calculation from part a.

Solution

a. Since solving for time involves dividing final velocity by acceleration, the vector notation should be dropped.

$a = 3.0\ \text{m/s}^2$

$v_i = 0$

$v_f = 11.0\ \text{m/s}$

$\Delta t = ?$

$$a = \frac{v_f - v_i}{\Delta t}$$

$$a = \frac{v_f}{\Delta t} \quad \because v_i = 0$$

$$\Delta t = \frac{v_f}{a}$$

$$= \frac{11.0\ \text{m/s}}{3.0\ \text{m/s}^2}$$

$$= 3.7\ \text{s}$$

It takes 3.7 s for the sprinter to reach his maximum velocity.

b. As the solution to part a. indicates, a higher rate of acceleration means that the sprinter will reach his maximum velocity in less time. This means that this athlete would spend more time racing at his maximum velocity, allowing him to finish the race sooner.

Example Problem 1.13

Figure B1.37: Sliding is a great social activity that makes exercise fun—why else would people happily climb a hill for hours at a time?

A group of teens riding an inflatable tube start from rest and travel down a hill, accelerating at an average rate of 1.15 m/s^2. Determine the speed reached by the teens after travelling for 6.0 s.

Solution

Since the velocity and acceleration vectors both point in the same direction and the question is asking for speed (a scalar), the vector notation is not necessary.

$v_i = 0$

$a = 1.15\ \text{m/s}^2$

$\Delta t = 6.0\ \text{s}$

$v_f = ?$

$$a = \frac{v_f - v_i}{\Delta t}$$

$$a = \frac{v_f}{\Delta t} \quad \because v_i = 0$$

$$v_f = a\Delta t$$

$$= (1.15\ \text{m/s}^2)(6.0\ \text{s})$$

$$= 6.9\ \text{m/s}$$

The teens reach a speed of 6.9 m/s after travelling down the hill for 6.0 s.

Practice

25. At a drag race, the dragste
a course about 400 m long
drove with an average acc
30.8 m/s^2, reaching a top s
of 500 km/h at the end of t
course. Assuming uniform
accelerated motion,
determine the time it
took the dragster to
reach its top speed.

Another Way to Think About Acceleration

Since acceleration is the change in velocity divided by the change in time, acceleration also occurs when an object is slowing down. What will be the shape of the graphs in this case? In the next investigation you will design an experiment to answer this question.

Investigation

Accelerated Motion—Slowing Down

Purpose

You will work with partners to design a procedure to answer the following questions:

- What is the shape of the position–time graph for an object that is slowing down?
- What is the shape of the velocity–time graph for this case?

Prediction

Make predictions to the questions posed in the purpose.

Materials

- ticker tape timer and ticker tape (or motion sensor)
- dynamics cart with a trigger-released, spring-loaded piston
- ramp
- masking tape
- metre-stick

Design

1. With your partner(s), decide how you will collect the data for this investigation. Be sure to discuss issues related to safety and the responsible use of the equipment. Concisely outline the procedure you intend to use.
2. Show your design to your teacher. Incorporate any suggested modifications; then check back with your teacher to get approval before continuing with the rest of the investigation.

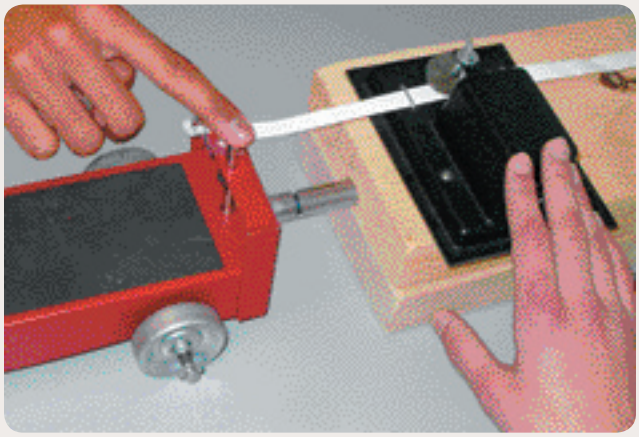


Figure B1.38: Here is a close-up of a dynamics cart with a trigger-released, spring-loaded piston.

Science Skills

- ✓ Initiating and Planning
- ✓ Performing and Recording
- ✓ Analyzing and Interpreting
- ✓ Communication and Teamwork

Procedure

Follow your procedure, and collect the necessary data. Use a table similar to the one on page 197.

Analysis

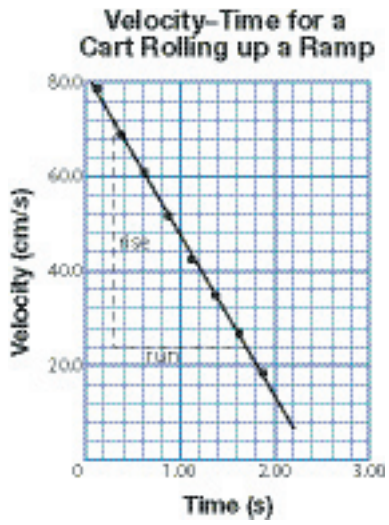
3. Calculate the average velocity and the midpoint time for each 0.25-s interval.
4. Use the first two columns of your table to plot a position–time graph. Draw a smooth curve that fits the data closely.
5. Use the last two columns of your table to plot a velocity–time graph. Use a ruler to draw a best-fit line.
6. Calculate the slope of the best-fit line on the velocity–time graph.
7. Describe the shape of the position–time graph. How does the shape of this graph describe the motion of the cart up the ramp?
8. Describe the shape of the velocity–time graph. How does the shape of this graph describe the motion of the cart up the ramp?

Evaluation

9. Do the results of this experiment support your hypothesis? Answer by comparing the predicted shapes of the two graphs with the actual shapes of the graphs.

Negative Acceleration

As you saw in the preceding investigation, "Accelerated Motion—Slowing Down," it is possible for a velocity–time graph to have a negative slope, resulting in a negative acceleration.



$$\text{acceleration} = \text{slope}$$
$$= \frac{\text{rise}}{\text{run}}$$
$$= \frac{24.0\ \text{cm/s} - 72.0\ \text{cm/s}}{1.70\ \text{s} - 0.30\ \text{s}}$$
$$= -34.3\ \text{cm/s}^2$$

The velocity values were all positive, but you can tell that the cart was slowing down because the acceleration was negative, indicating that it was pointing in the opposite direction. An object will slow down if the acceleration and velocity vectors point in opposite directions. An object will speed up if the velocity and acceleration vectors point in the same direction. As you'll see in Example Problem 1.14, a negative acceleration can mean that an object is speeding up if the velocity values are also negative.

Example Problem 1.14

Assuming the effects of air resistance are ignored, objects accelerate at 9.81 m/s^2 downward when they fall through the air near Earth's surface. This means that if a toy car falls from the flat roof of a five-storey building, it would travel the 15-m distance in about 1.75 s. Use the sign convention of down being the negative direction and up being the positive direction as you answer the next questions.

continued

a. If the car started from rest, determine the magnitude and direction of the car's final velocity after 1.75 s (just before striking the ground). State your answer in metres per second and kilometres per hour.

b. In this situation, the car was speeding up but the acceleration was negative. Sketch a graph of the motion to help explain how it is possible to have a negative acceleration in a situation where the object is speeding up.

Solution

a.

$$\vec{a} = 9.81\ \text{m/s}^2\ [\text{down}]$$
$$= -9.81\ \text{m/s}^2$$

up +
down -

$$\Delta\vec{d} = 15\ \text{m}\ [\text{down}]$$
$$= -15\ \text{m}$$

$$\Delta t = 1.75\ \text{s}$$

$$\vec{v}_i = 0$$

$$\vec{v}_f = ?$$

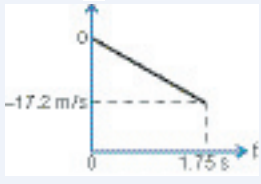
$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$\vec{a} = \frac{\vec{v}_f}{\Delta t} \quad \because v_i = 0$$

$$\vec{v}_f = \vec{a}\Delta t$$
$$= (-9.81\ \text{m/s})(1.75\ \text{s})$$
$$= -17.2\ \text{m/s}$$
$$= -17.2\ \frac{\text{m}}{\text{s}} \times \frac{1\ \text{km}}{1000\ \text{m}} \times \frac{3600\ \text{s}}{1\ \text{h}}$$
$$= -61.8\ \text{km/h}$$

The car's final velocity after 1.75 s is 17.2 m/s[down] or 61.8 km/h[down]. **Note:** The displacement value was provided but not needed in the solution.

b.



The acceleration value of -9.81 m/s^2 means that for every second, the car was going 9.81 m/s faster in the downward direction.

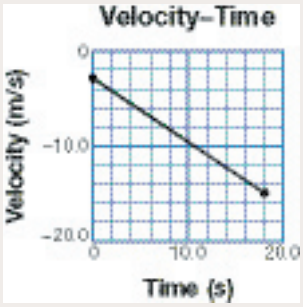
-9.81 m/s^2 = 9.81 m/s/s[down]

So, as the car fell, its velocity became more negative. In other words, it was speeding up in the negative direction.

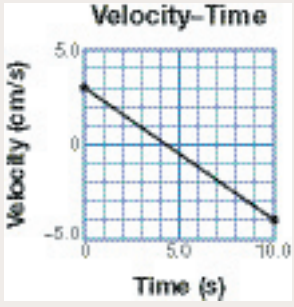
Practice

26. A jet maintains an average acceleration of 2.20 m/s^2 for 36.0 s to reach the speed required for takeoff. If the jet starts from rest, determine its take-off speed in kilometres per hour.
27. Determine the acceleration from the information provided in each of the following graphs.

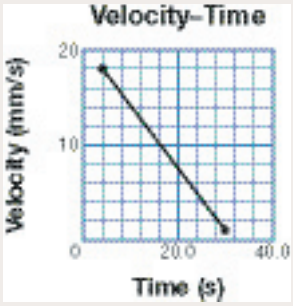
a.



b.



c.



d.

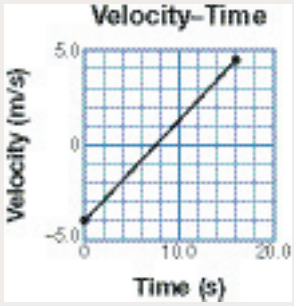
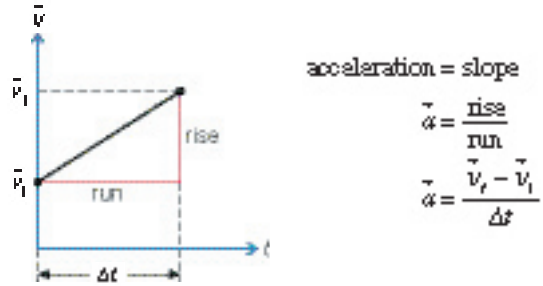


Figure B1.39: Larger aircraft require longer runways.

28. A motorist travelling on a highway sees a moose on the road and begins to brake. At the instant the braking begins, the velocity of the vehicle is 100 km/h, north, and the vehicle stops 5.0 s later. Calculate the magnitude and direction of the acceleration of the vehicle in m/s^2.
29. Refer to the information in Example Problem 1.14. If you were driving a car near the edge of the roof of this building, you would take great care not to go over the edge because of the possibility of a high-velocity impact with the ground. Yet, some drivers do not seem to have the same degree of caution when they are operating motor vehicles on the highway at this same velocity, even though the impact with an object (like a concrete pillar supporting an overpass) is just as hazardous. Refer to the information from earlier in this chapter to explain this behaviour.

1.5 Summary

Acceleration is equal to the slope of a velocity–time graph. The following shows how the equation for acceleration is derived from the definition of slope applied to this graph.



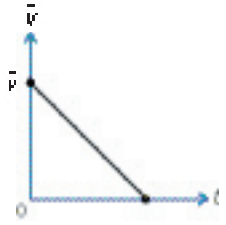
$$\text{acceleration} = \text{slope}$$

$$\vec{a} = \frac{\text{rise}}{\text{run}}$$

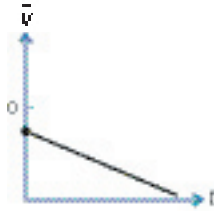
$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

If the slope of the velocity–time graph is negative, the acceleration is negative. Negative acceleration can have different meanings.

Negative Acceleration

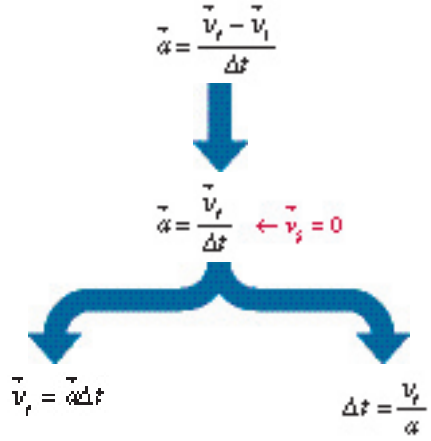


When $\vec{v}_i$ is positive, negative acceleration means slowing down because the velocities are approaching zero.



When $\vec{v}_i$ is negative or zero, a negative acceleration means speeding up because the velocities are becoming larger negative values.

For situations where the initial velocity is zero, the acceleration equation can be rearranged into the following:



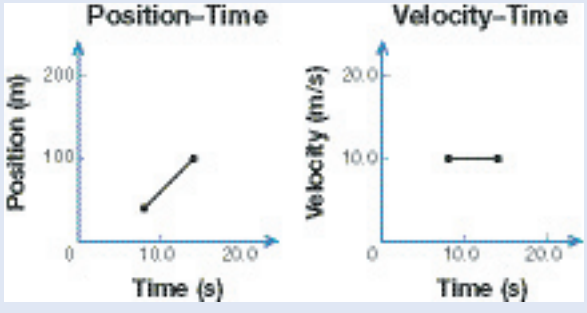
Notice how the vector notation is dropped when determining time. The vector notation should also be dropped when velocity and acceleration vectors point in the same direction or when the question asks you to solve for a scalar quantity, like speed.

1.5 Questions

Use the following information to answer questions 1 to 3.

Recall the description on page 195 of the car travelling between two intersections. The car accelerated from rest and travelled 40 m in 8.0 s. The car then travelled an additional 60 m at a constant speed of 10.0 m/s for 6.0 s. From there, the car travelled a final 25 m in 5.0 s as it slowed to a stop for the next red light.

The following sketches show only the part of motion when the car was travelling at a constant speed.



Knowledge

1. Copy the position–time graph into your notebook.
   - **a.** Add to the graph by sketching the shape of the graph in the first 8.0 s.
   - **b.** Add to the graph by sketching the shape of the graph in the last 5.0 s.
2. Copy the velocity–time graph into your notebook.
   - **a.** Add to the graph by sketching the shape of the graph in the first 8.0 s.
   - **b.** Add to the graph by sketching the shape of the graph in the last 5.0 s.

Applying Concepts

3. Refer to the velocity–time graph.
   - **a.** Determine the acceleration of the car in the first 8.0 s.
   - **b.** Determine the acceleration of the car in the last 5.0 s.
   - **c.** The description of the motion states that the car travelled 40 m in the first 8.0 s. Use the values from the velocity–time graph to confirm that the displacement was in fact 40 m. (**Hint:** Displacement is equal to the area under a velocity–time graph.)
   - **d.** The description of the motion states that the car travelled 25 m in the last 5.0 s. Use the values from the velocity–time graph to confirm that the displacement was in fact 25 m.